Reliability assessment of an aerospace component subjected to fatigue loadings: APPRoFi project

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Abstract. This paper presents an overview of the methodology implemented during the APPRoFi project to assess the reliability of a studying case called "blade support" under fatigue loadings. The structure is defined by a deterministic geometry, a random strength derived from fatigue tests performed on structures and specimens and an elastic-plastic material whose parameters are random variables. A random time variant displacement load is applied to the structure. It is characterised using nine spectra and a strategy to generate virtual rainflow matrices. This strategy defines the scatter observed on the nine empirical cumulative distribution functions of the displacements to create new spectra. Finally, the reliability of the studying case is assessed using a numerical method called AK-IS based on Kriging and Importance Sampling. It is used to obtain an accurate estimation of the failure probability for a small computational effort.

1 INTRODUCTION

The assessment of reliability is nowadays a major challenge in the design of industrial structures. Codified deterministic methods are employed in industry to design structures subjected to fatigue failure. However, these methods do not give information about the probability of failure of the structures or about the most important factors on the design. To obtain an accurate idea of the probability of failure and to efficiently design a structure without endangering its integrity, probabilistic approaches should be used.

In industry such as the aerospace one, the probabilistic approaches have to be used to give information about the probability of critical situations. For this reason, French public laboratories (IFMA-LaMI, ENS-LMT CACHAN, UTC-LABORATOIRE ROBERVAL) and companies (CETIM, SNECMA, PHIMECA, MODARTT) considered as experts in their domains of interest have gathered to create the APPRoFi project – APproche mécano-Probabiliste pour la conception Robuste en Fatigue". Funded by ANR – Agence Nationale de la Recherche – this project is based on the application of probabilistic approaches to assess the reliability of two industrial studying cases subjected to fatigue crack initiation. The project aims at demonstrating the interests of such probabilistic approaches for industry.

In this paper, the methodology implemented in the project is used to assess the reliability of a structural component. The methodology is outlined Section 2. Following this, the issue of rare events probability assessment is introduced Section 3.1. An innovative reliability method called AK-IS is detailed Section 3.2. It enables to assess for a relatively low computational effort, the small failure probabilities of time-consuming performance functions. The structure is introduced Section 4.1. The available data enabled to define:

- random fatigue loadings detailed Section 4.2,
- random material based on the Chaboche's law presented Section 4.3,
- random strength distribution using a probabilistic fatigue curve, described Section 4.4.

Following this, the reliability is assessed Section 4.5.

2 PROBABILISTIC APPROACH DEVELOPED IN APPROFI

The fatigue approach developed in APPRoFi project is inspired by the probabilistic Stress-Strength method [1] which is based on the well-known Stress-Strength Interference (SSI) analysis [2] and the equivalent fatigue theory [1]. Given a fatigue time-dependent load $F(t)$ applied to a structure, the first step of the probabilistic Stress-Strength method consists in quantifying the fatigue load variability. The load scatter observed in service is modelled by elementary life situations which enable then to generate virtual loads. For each of these loads, an equivalent in damage scalar load F_{eq} repeated N_{eq} times is computed. It is called the equivalent fatigue load. By generating numerous loads, the equivalent fatigue load variable $F_{eq}(N_{eq},\omega)$, function of the number of cycles N_{eq} and the random ω, can be modelled according to a known distribution (such as Gaussian, lognormal or Weibull for example). The second step consists in the SSI analysis which defines the performance function G as the difference between the structural strength at N_{eq} cycles $R(N_{eq},\omega)$ obtained from fatigue tests and the equivalent fatigue load. The failure probability P_f which corresponds to the probability that the strength is lower than the stress is then derived from analytical formulas or numerical integration using SSI. This method is well adapted in an industrial context for pre-dimensioning in the field of high cycle fatigue with linear mechanical behaviour. However, it may be inaccurate or more difficult to use for non-linear cases and complex variability of the loading severity that cannot be modelled with classical distributions. In addition, the approach gathers geometrical and material uncertainties in the strength side of the problem and their effects are not quantified separately on the reliability.

The approach developed in APPRoFi attempts to solve some of these issues. First, the equivalent fatigue load is not directly considered as a random variable with a known distribution. It is preferred to characterise parameters determining the applied load as random variables. The load simulated through these random parameters is then defined as a combination of random variables which may not result in a Gaussian distribution. Second, the fatigue tests results are used to determine a probabilistic fatigue curve rather than only the structural strength at N_{eq} cycles. Following this, a structural behaviour is created by selecting a fractile curve to calculate the strength value but also the equivalent fatigue stress (which requires the knowledge of a fatigue curve).

The approach developed in APPRoFi is presented Figure 1. It is described with only random material parameters but it may also involve geometrical variables. On the right hand-side of the figure, the structural strength is calculated according to the probabilistic fatigue curve. The left part of the figure shows the calculation of the equivalent fatigue stress at N_{eq} cycles which depends on the loading parameters but also on the fractile of the probabilistic fatigue curve. Following this, the response of the mechanical model is obtained as the equivalent fatigue stress at N_{eq} cycles. To assess the reliability of the structure defined by its performance function G, several time demanding computations of the mechanical model are required. To reduce the computational effort, metamodels (or response surface method) can be a solution to approximate the performance function thanks to a few computed configurations. In this case, a new method involving a Kriging metamodel and called AK-IS (described in Section 3.2) is developed to monitor the few samples to compute on the mechanical model in order to assess the failure probability of the structure.

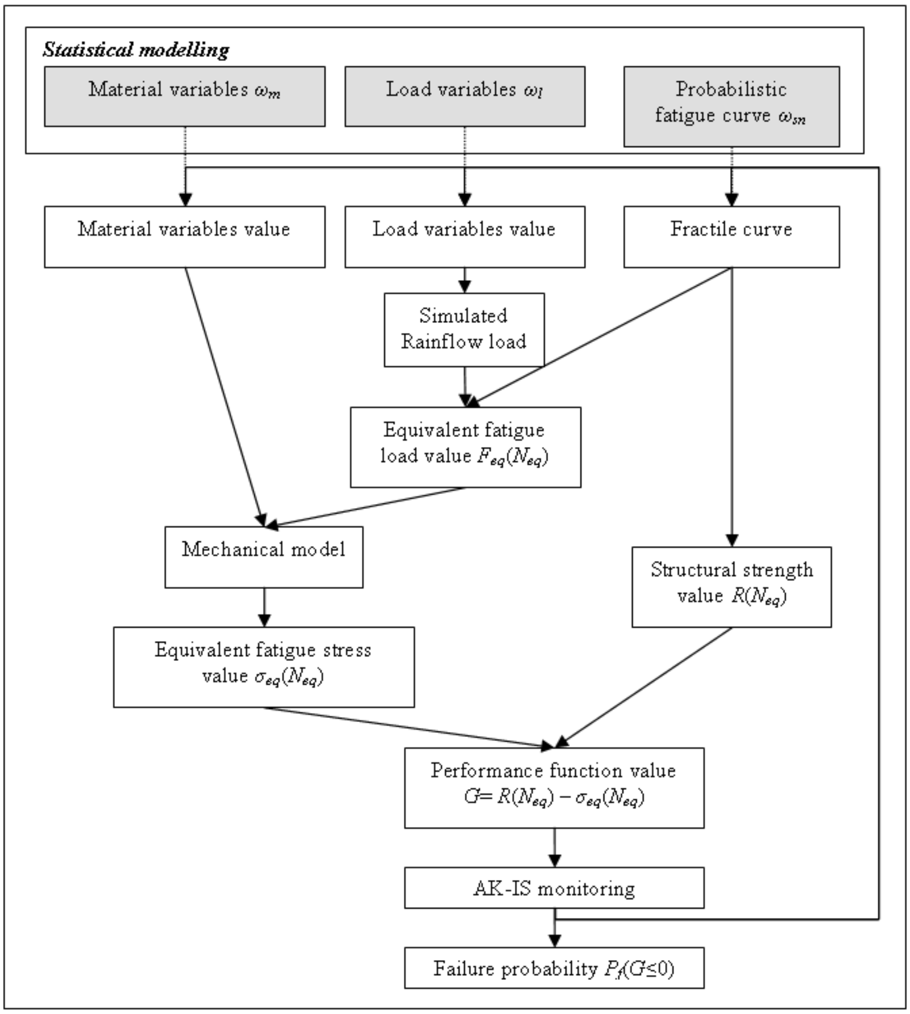

Figure 1: Flowchart of the probabilistic approach developed in APPRoFi.

3 RELIABILITY ASSESSMENT OF RARE EVENTS

3.1 Assessment of the failure probability by simulations

Given the n-dimensional space R^n, the random variables of the problem are noted X_1 to X_n and a sample of this space $\mathbf{x}=\{x_1,\ldots,x_n\}$. Structural reliability aim is to obtain the failure probability P_f, *i.e.* the probability of being in the failure domain F or $G(\mathbf{x})\leq 0$ [3,4]. Such probability can be written:

$$P_f = \int_F f_{X_1,\ldots,X_n}(x_1,\ldots,x_n)dx_1\ldots dx_n \tag{1}$$

where $f_{X_1,\ldots,X_n}$ corresponds to the joint probability density function (joint p.d.f.).

By introducing I_F the failure counter which is equal to {0 if $G(\mathbf{x})>0$ and 1 if $G(\mathbf{x})\leq 0$}, the integral becomes:

$$P_f = \int_{\mathrm{R}^n} I_F\ f_{X_1,\ldots,X_n}(x_1,\ldots,x_n)dx_1\ldots dx_n = \mathrm{E}\left[I_F\right] \tag{2}$$

where E[ ] is the mathematical expectation.

Simulation methods are often used to estimate the failure probability. The most famous, Monte Carlo Simulation (MCS), consists in simulating a large number of samples and computing them on the performance function, see Figure 2. By noting the simulated samples $\mathbf{x}^{(j)}$ for $j=1,\ldots, M$, the failure probability can be estimated by:

$$P_f \approx \hat{P}_f = \frac{1}{M}\sum_{j=1}^{M} I_F(\mathbf{x}^{(j)}) \tag{3}$$

The coefficient of variation δ defines a confidence interval on the failure probability. An estimate of such coefficient is given by:

$$\delta = \sqrt{\frac{1-\hat{P}_f}{M\ \hat{P}_f}} \tag{4}$$

MCS is very convenient to assess reliability. However, the samples are generated around the mean value of the random variables and rare events are then difficult to simulate as a large population has to be generated. For the practical application of this paper, failure is an extremely rare event and therefore its probability is very small. MCS cannot be applied on such case and it is then necessary to simulate samples not around the mean values of the random variables but towards the failure domain [4]. Methods based on this statement are called variance reduction techniques. The Importance Sampling (IS) method [5] is one of them. It consists in concentrating the sampling in the vicinity of the most probable failure point noted P^* as depicted Figure 3. By this way, the conditional failure event is less rare and the number of samples remains low. By noting the joint p.d.f. of the new sampling $h_{X_1,\ldots,X_n}$, the integral to be evaluated is then given as:

$$P_f = \int_{\mathrm{R}^n} I_F\ \frac{f_{X_1,\ldots,X_n}(x_1,\ldots,x_n)}{h_{X_1,\ldots,X_n}(x_1,\ldots,x_n)} h_{X_1,\ldots,X_n}(x_1,\ldots,x_n)dx_1\ldots dx_n \tag{5}$$

The joint p.d.f. $h_{X_1,\ldots,X_n}$ is often taken in the standard space as the Gaussian distribution centred on the most probable failure point with standard deviation equal to 1. The samples simulated through it are denoted $\tilde{\mathbf{x}}^{(j)}$ for $j=1,\ldots,Q$ with Q much lower than M. The integral is then estimated by:

$$P_f \approx \tilde{P}_f = \frac{1}{Q}\sum_{j=1}^{Q} I_F(\tilde{\mathbf{x}}^{(j)})\frac{f_{X_1,\ldots,X_n}(\tilde{\mathbf{x}}^{(j)})}{h_{X_1,\ldots,X_n}(\tilde{\mathbf{x}}^{(j)})} \tag{6}$$

The coefficient of variation δ can be obtained by the ratio between the failure probability standard deviation and the estimated failure probability. The standard deviation $\tilde{\sigma}_{\mathrm{IS}}$ reads:

$$\tilde{\sigma}_{\mathrm{IS}}^2 = \frac{1}{Q-1}\left(\frac{1}{Q}\sum_{j=1}^{Q}\left(I_F(\tilde{\mathbf{x}}^{(j)})\left(\frac{f_{X_1,\ldots,X_n}(\tilde{\mathbf{x}}^{(j)})}{h_{X_1,\ldots,X_n}(\tilde{\mathbf{x}}^{(j)})}\right)^2\right)-\tilde{P}_f^2\right) \tag{7}$$

Such method gives satisfactory results when the weight of the failure probability is located in the vicinity of the most probable failure point. However, the performance function may require the computation of a time-consuming numerical model and such simulation methods may still require too many samples to be computed. A solution to this issue can be to couple these simulation methods with response surface method (or metamodels). This is the case of the AK methods with Kriging.

3.2 AK-IS method

To reduce drastically the number of computations, the reliability method AK-MCS [6] is developed. It is an Active learning method combining Kriging metamodel and Monte Carlo Simulation. It consists in classifying (into the failure and safe groups) a Monte Carlo population without evaluating all the samples on the performance function. Based on a few known samples (the design of experiments), the Kriging metamodel predicts the performance function value $\mu_{\hat{G}}(\mathbf{x})$ at any sample $\mathbf{x}$ and also provides an estimation of the uncertainty on this value thanks to the Kriging variance. Thanks to the Kriging properties, AK-MCS can classify a population with only a few known samples and can identify the samples which should be "learnt" to improve the classification. By computing these samples, the metamodel response is improved iteratively until the belonging to the safe or the failure group of all the samples is certain enough. It has been shown to be very parsimonious on several examples. However, the prediction by Kriging of a large population can become computer-demanding and a modification to deal with rare events should be considered.

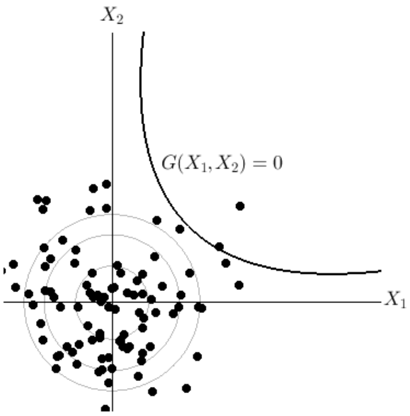

Figure 2: Monte Carlo sampling.

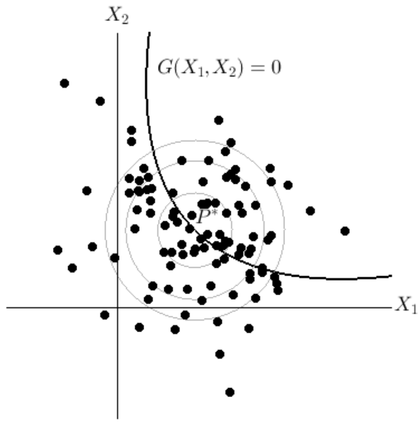

Figure 3: Importance sampling.

Precisely, this paper proposes to improve AK-MCS by combining Kriging and the IS technique to deal with rare events. The method is called AK-IS. Like IS, it is composed of two very distinct steps. The first one consists in finding the coordinates of P^* using the Hasofer-Lind-Rackwitz-Fiessler (HLRF) algorithm [7]. The second step is the classification on the principles of AK-MCS of a population of Q samples noted $\tilde{\mathbf{x}}^{(j)}$ for j=1,…,Q centred on the point found previously. By noting the Kriging predicted failure domain $\hat{F}$, the failure probability is approximated by (6) using the Kriging performance function values $\mu_{\hat{G}}(\tilde{\mathbf{x}}^{(j)})$.

4 STUDYING CASE

4.1 Introduction

A simplified model of the blade support structure is used in this project. A displacement load is applied to it.

For this project, the available data are composed of:

- 9 spectra giving the time evolution of displacement.
- 3 hysteresis loops performed for different strain levels to characterise the Chaboche's law of the material,
- 15 fatigue tests monitored in displacement and performed on real structures,
- 65 fatigue tests monitored in strain and conducted on specimens.

In relation with the available data, the choice is made to consider five parameters as random variables to assess the reliability of the structure: the fatigue load, 3 parameters of the elastic-plastic material, and the structural strength.

4.2 Fatigue loading

To characterise the displacement observed by the structure, 9 spectra which give the evolution of displacement (Figure 4) are used. Cycles (mean / amplitude) are extracted from the time-dependent displacement using a Rainflow procedure. An example is given Figure 5.

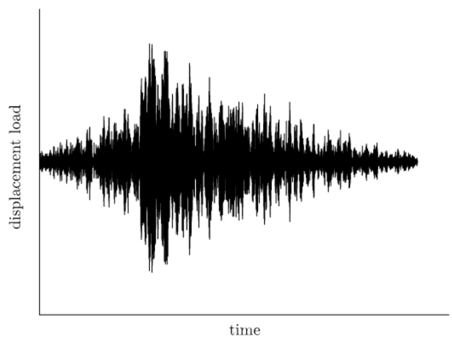

Figure 4: Evolution of a displacement spectrum.

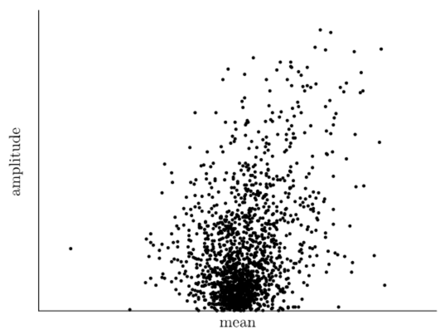

Figure 5: Example of a displacement spectrum converted into Rainflow cycles.

The approach to model the load scatter is detailed as follows. First, the Gerber parabola is used to convert the cycles with variable load ratios into cycles with a load ratio equal to -1, *i.e.* cycles with symmetric amplitudes and mean zero. This transformation permits to define the displacement load as a 1-dimensional load that is to say the displacement symmetric amplitude a'. Then, the 9 empirical cumulative distribution functions (c.d.f.) of the displacement amplitudes, Figure 6, are considered. The amplitude scatter observed for a level m of the c.d.f. is defined as a Gaussian distribution $a'^{(m)}(\omega)$ with the empirical mean $\mu_{a'}^{(m)}$ and standard deviation $\sigma_{a'}^{(m)}$ calculated on the 9 spectra. A random c.d.f. can be generated thanks to a Gaussian variable $C_S(\omega)$ of mean zero and unit standard deviation. This random variable is known as the coefficient of severity and by selecting a value c_S, it defines the fractile to use at any level m of the c.d.f.:

$$a'^{(m)}(c_S) = \mu_{a'}^{(m)} + c_S\ \sigma_{a'}^{(m)} \tag{8}$$

An example of a generated c.d.f. is given Figure 7 for c_S=2.5. The Gaussian distributions for three different levels (a, b, c) are also depicted in the figure. The parameter c_S defines the fractile of each distribution which then determines the load c.d.f..

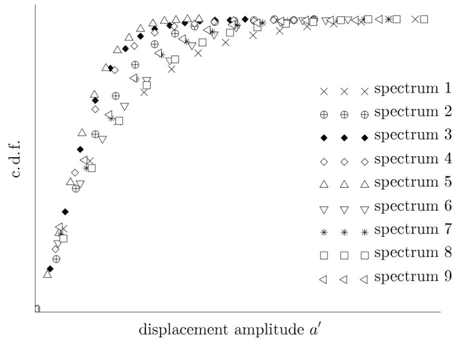

Figure 6: Cumulative distribution functions of the nine spectra.

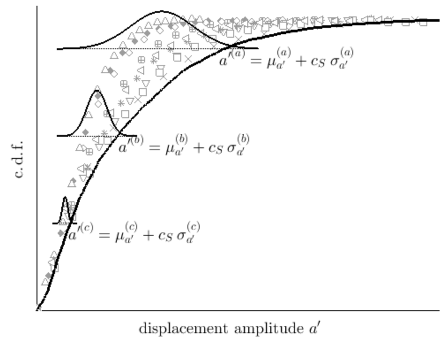

Figure 7: c.d.f. generated with c_S= 2.5, a, b and c representing 3 different levels.

Finally, the displacement load is generated by discretising the c.d.f. into a number of cycles similar to the one observed in the 9 spectra. The displacement amplitudes obtained at these cycles define the random load which is then transformed into an equivalent fatigue displacement noted u_{eq} to apply to the numerical model. The modelling is validated Figure 8 by comparing the equivalent fatigue displacement of the 9 spectra with 1,000 simulated.

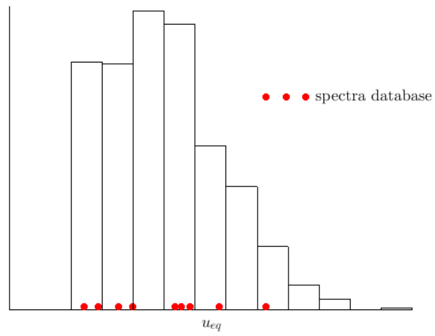

Figure 8: Distribution of 1,000 equivalent fatigue displacements and the 9 equivalent fatigue displacements from the spectra database.

4.3 Material parameters

The material behaviour is elastic-plastic that can be characterised by the Chaboche's law [8]. Three parameters of this law are taken as random variables. These parameters are the yield strength $K(\omega)$, the hardening parameters $C(\omega)$ and $D(\omega)$. Given the stress range $\Delta\sigma$ and plastic strain range $\Delta\varepsilon_p$, the Chaboche's law can be formulated as:

$$\frac{\Delta\sigma}{2} = K(\omega) + \frac{C(\omega)}{D(\omega)} \tanh\left(D(\omega) \frac{\Delta\varepsilon_p}{2} \right) \tag{9}$$

Bayesian inference is used to update hypothetical distributions for the material parameters. This is done thanks to 3 hysteresis loops performed for different strain levels. The parameters are in the end defined as lognormal variables and C and D are considered to be strongly correlated.

4.4 Probabilistic Fatigue curve description & strength distribution

To define the strength distribution $R(N_{eq},\omega)$ at N_{eq} cycles, fatigue data on 15 real structures and on 65 specimens are available. The fatigue curve is given as the Smith Watson Topper stress σ_{SWT} [9] versus the number of cycles. The Smith Watson Topper stress reads:

$$\sigma_{\mathrm{SWT}} = \sqrt{E\,\sigma_{\max}\,\frac{\Delta\varepsilon_t}{2}} \tag{10}$$

where E is the Young modulus, $\sigma_{\max}$ the maximum stress and $\Delta\varepsilon_t$ the strain range.

The fatigue curve trend is defined as a Basquin curve and the Smith Watson Topper stress for a set number of cycles N is considered to be a Gaussian random variable. Furthermore, its coefficient of variation is defined as constant at any N. Figure 9 depicts the probabilistic fatigue curve and the strength distribution $R(N_{eq},\omega)$, *i.e.* the Smith Watson Topper distribution at N_{eq} cycles.

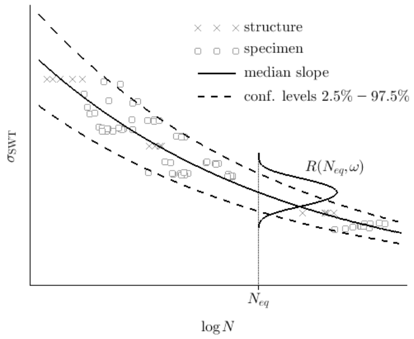

Figure 9: Probabilistic fatigue curve based on specimens and structures tests.

4.5 Reliability assessment

The performance function noted G is formulated using the Smith Watson Topper stress. It consists of the difference between the strength $R(N_{eq},\omega)$ defined previously and the numerical model output, *i.e.* the maximum Smith Watson Topper stress $\sigma_{\mathrm{SWT}}(N_{eq},C_S(\omega),k(\omega),C(\omega),D(\omega))$ obtained from the applied equivalent fatigue displacement at N_{eq} cycles:

$$G = R\left(N_{eq},\omega\right) - \sigma_{\mathrm{SWT}}\left(N_{eq},C_S(\omega),K(\omega),C(\omega),D(\omega)\right) \tag{11}$$

Evaluating the performance function requires the computation of a very time consuming numerical model. As the assessment of the failure probability may require numerous runs, efforts are made to reduce the computational cost in two ways. First, the LATIN method [10] is used to reduce the computational cost of successive evaluations by stating that 2 runs with different sets of parameters may have some similarities. Second, the AK-IS method, introduced Section 3.2, is implemented to identify the most relevant sets of parameters to run in order to assess the failure probability with accuracy.

The flowchart of the APPRoFi probabilistic approach specific to this case study is detailed Figure 10. The first step of AK-IS consists in finding the most probable failure point. For this study case, the HLRF algorithm [7] requires 3 iterations to obtain it. An iteration is composed of 6 runs of the performance function. The number of calls N_{call} to G is then 18+1=19. However, as the calculation of the gradient in the strength direction does not require a new computation of the numerical model, the number of runs is only of 16. The reliability index β [4] is equal to 5.664 which gives a FORM approximation of the failure probability of $\Phi(-\beta)=7.408\times10^{-9}$ with Φ the standard Gaussian c.d.f. [4]. The coordinates noted $\left\{u_R^*,u_{C_S}^*,u_K^*,u_C^*,u_D^*\right\}$ of the most probable failure point in the standard space can be used to measure the importance of each random variable parameters on the structural reliability. In this paper, focus is made on the elasticity e_{σ_i} (normalised sensitivity) of the reliability index to the i^{th} random variable standard deviation σ_i which is given as:

$$e_{\sigma_i} = \frac{\sigma_i}{\beta}\left.\frac{\partial\beta}{\partial\sigma_i}\right|_{u^*} \tag{12}$$

It can be shown [4] that in case of Gaussian independent random variables, the elasticity is equal to:

$$e_{\sigma_i} = -\alpha_i^2 = -\left(-\frac{u_i^*}{\beta}\right)^2 \tag{13}$$

where α_i is the direction cosine of the i^{th} standardised random variable.

In this paper, material variables are not Gaussian and independent, however, the formulation in (13) gives a sensible approximation of the elasticities. They are given Table I. It is shown that the material parameters in this case have little impact on the reliability. In fact, the structure remains in the elastic for most applied displacements. Therefore, C

and *D* have very rarely an impact on the mechanical behaviour of the structure. In addition to that, it is shown that the strength standard deviation is the parameter which has the most influence on the structural reliability.
The second step of AK-IS consists in the classification of a population centred on the most probable failure point found previously. The population to classify is composed of 10^4 samples. The results of this step are given Table II. Step 2 requires 11 calls to the performance function to classify the population. Consequently, only 30 runs are performed to assess the failure probability. The limit state seems to be rather linear as the FORM approximation and the AK-IS results are very close. In this case, the second step of AK-IS brings, for a small additional computational demand, further information that enables to confirm the failure probability found using FORM approximation.

Table I: Elasticities of the reliability index to the different random standard deviations.

Strength	Load		Material	
e_{σ_R}	$e_{\sigma_{C_S}}$	e_{σ_K}	e_{σ_C}	e_{σ_D}
-0.729	-0.270	-2.5×10^{-5}	-1.6×10^{-5}	-0.4×10^{-5}

Table II: AK-IS results.

N_{call} (Step 1 + Step 2)	$\tilde{P}_f$	δ
19 + 11	7.46×10^{-9}	2.50%

5 CONCLUSION

In this paper, a probabilistic method to assess the reliability of structures subjected to fatigue loadings is presented. It is based on the stress-strength interference analysis and the equivalent fatigue theory. Five random variables are considered for the studied case: three for the non-linear material properties, one for the displacement load and one for the structural strength. To assess the reliability of the structure, several runs of the mechanical model are required. This model being a finite element one, it is consequently computer-demanding. Two time reduction techniques are used to obtain the failure probability in an acceptable time. First, the LATIN method is used to run in a faster and more efficient way successive sets of parameters. Second, the AK-IS reliability method based on the Kriging metamodel is implemented to select parsimoniously the sets of parameters to run on the finite element model in order to obtain an accurate assessment of the failure probability.

The method proposed in this paper shows promising results in terms of computation time and reliability assessment. In fact, it shows some advantages compared to the classic stress-strength interference analysis. First, no assumption is made on the statistical distribution of the stress. Second, information about the importance of the model inputs enables to determine which random variables or random variable parameters have the most significant impacts on the failure probability.

However, such method being based on the equivalent fatigue theory requires the assumption of a linear relation between the load applied and the numerical model output. For non linear cases, the proposed method may lead to errors in the reliability assessment. Therefore, the performance function should be formulated in damage rather than stress so to remove the equivalent fatigue step [11]. Nevertheless, such formulation would lead to a much larger computational effort which could be reduced by using metamodel such as Kriging not to approximate directly the performance function of the structure but the numerical model output. The Rainflow load should then be directly applied on the metamodel and Miner's rule should be used to evaluate the damage. In the end, by considering that a damage superior to 1 corresponds to failure, the structural reliability can be assessed by coupling the metamodel with Monte Carlo simulations or Importance Sampling.

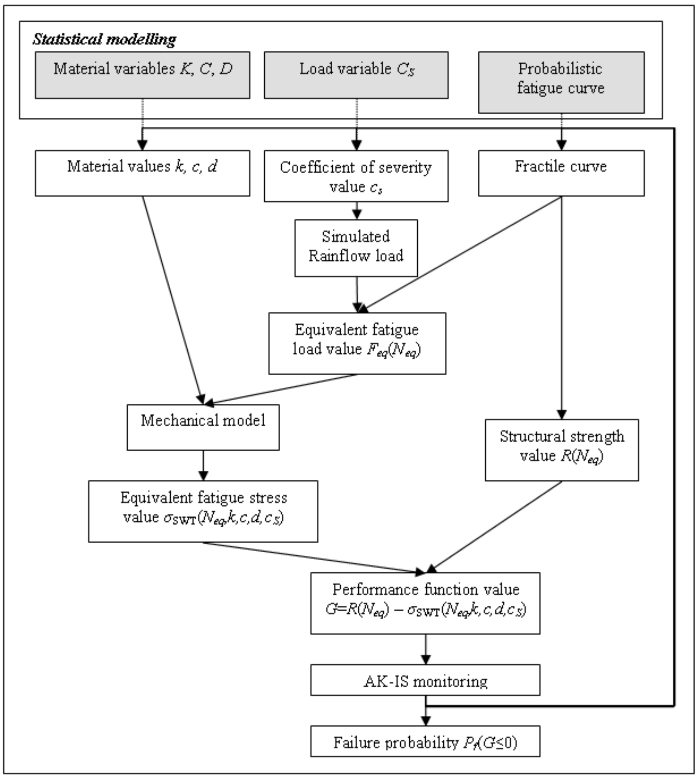

Figure 10: Flowchart of the APPRoFi probabilistic approach for the case study.

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